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Finite-time stochastic input-to-state stability and observer-based controller design for singular nonlinear systems*

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Abstract. This paper investigated observer-based controller for a class of singular nonlinear systems with state and exogenous disturbance-dependent noise. A new sufficient condition for finite-time stochastic input-to-state stability (FTSISS) of stochastic nonlinear systems is developed. Based on the sufficient condition, a sufficient condition on impulse-free and FTSISS for corresponding closed-loop error systems is provided. A linear matrix inequality condition, which can calculate the gains of the observer and state-feedback controller, is developed. Finally, two simulation examples are employed to demonstrate the effectiveness of the proposed approaches.

Keywords: finite-time stochastic input-to-state stability, observer, stochastic nonlinear systems, Brownian motion.

1 Introduction

Singular systems can naturally describe the real systems than regular systems because of the fact that singular systems can better preserve the structure of some physical systems and impulsive elements [5, 9, 13, 22, 23, 30]. This class of systems are described by

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a differential-algebraic equation. They are encountered in many scientific areas such as circuit systems, electrical networks, economic systems and so on [6, 8]. Based on the system behavior, many researchers are attracted and many results of normal systems have been successfully extended to singular systems. However, most of the existing results are derived under these conditions that the singular systems are linear systems. When we consider the practical control systems, for example, mechanical systems, economics, they often exhibit strong nonlinear dynamics. The linear singular systems models can not meet the demand when we want to analyze or control these systems. Taking the advantages of both singular systems and nonlinear systems [21], singular nonlinear systems model can be used to describe physical processes more conveniently and accurately and also permit to better model systems. Many phenomena, such as network models, earthquakes and population evolution, can be described with this class of models. Thus the singular nonlinear systems have attracted attention in the control and observation theory, and related research results have been published [14, 19, 25, 31]. In particular, in the past few decades, considerable attention has been devoted to singular nonlinear systems with Brownian motions due to their extensive applications in biological systems, mechanical systems, economics and other areas. A variety of works have been published with respect to the stability, sliding mode control and filtering problems of singular nonlinear systems with Brownian motions [27, 32, 40]. As well know, high cost of measuring devices and other technical limitations lead to unavailability of system states in most of control applications. However, state feedback control, which is utilized to achieve the excellent closed-loop system performance, is invalid in this case. To overcome this problem, observers are used to estimate the unmeasured states [2, 3, 12, 26, 36]. Observer-based controllers are often used to improve the system performances or stabilize unstable systems [10, 29]. In [10], observer-based controller for the descriptor system with Brownian motions was investigated. A sequential design technique is proposed to calculate the control and observer gains by solving linear matrix inequalities. In [29], a new H_∞ reduced-order observer-based controller synthesis structure is investigated for nonlinear systems.

It is well known that the term of input-to-state stability (ISS) plays an important role in stability analysis and controller design of nonlinear systems [4, 7, 16, 17, 34]. Input-to-state stability, which was originally proposed for deterministic continuous-time systems by Sontag in 1980s [24], has become a central concept in nonlinear controller design and analysis. As the natural extension of ISS, integral input-to-state stability (iISS), input-output-to-state stability (IOSS) and stochastic input-to-state stability (SISS) have been made for different types of dynamical systems [28, 37].

Finite-time stability, which is an important field of stability property, has attracted particular interests of researchers because of its significance in applications and theoretical research. Different from the classical Lyapunov stability and exponential stability, which describe the behavior of the trajectories of a dynamical system in an infinite-time interval, finite-time stability concerns the stability of a system over a finite interval of time. In many practical problems, there are systems, which work in a short time, for example, network system, missile system, vehicle maneuvering, robotics. Hence, it is more valuable that the convergence of a dynamical system is realized in finite time rather than infinite time. Such a necessity may arise either when the system is only defined over a finite interval of time or when it is defined for all time, but the performance of its trajectory is only interest over a finite time interval. Finite-time stability can be classified into two categories. One can be described as follows: the system states do not exceed a certain bound during a specified time interval under a given bound on initial conditions [15, 18]. The other is defined as the system states reach the system equilibrium in a finite time [11, 35, 38]. Here we shall focus our attention on the latter case. In [11], finite-time uniform stability of functional differential equations with applications in network synchronization control was investigated. In [35], an important Lyapunov theorem on finite-time stability for stochastic nonlinear systems was established, and a Lyapunov theorem on finite-time instability was also proved. The problem of global finite-time stabilization in probability for a class of stochastic nonlinear systems was investigated in [38]. As a composite concept, FTSISS has attracted much attention, and some efficient results have been derived [1, 42]. In [1], the finite-time stochastic input-to-state stability problem for a class of impulsive switched stochastic nonlinear systems was investigate. In [42], the problems of finite-time globally asymptotical stability in probability and finite-time stochastic input-to-state stability for switched stochastic nonlinear systems were investigated. Some new definitions and some sufficient conditions on finite-time globally asymptotical stability in probability and finite-time stochastic input-to-state stability were given. To the best of the authors' knowledge, the results on controller for the stochastic singular systems with Brownian motions mostly focus on linear systems or Lipshitz nonlinear systems. This motivates us to develop the research on a more general stochastic singular nonlinear systems.

Based on the views above, this paper focuses on the observer-based control problem for a class of stochastic singular nonlinear systems with Itô-type stochastic disturbance. For this class of stochastic singular nonlinear systems, there are almost no FTSISS result in the open literature. A new sufficient condition for FTSISS is developed. At the same time, the dynamics considered here covers a broad family of nonlinear systems. In some real systems, systems states are not completely measurable, therefore, the observer-based controller are need in this case. In this paper, a new design method is proposed to design the controller gain and observer gain of the observer-based controller for a class of stochastic singular nonlinear systems by solving linear matrix inequalities. Finally, simulation examples are given to illustrate the effectiveness of our results.

Notations. The symbol $\mathbb{R}$ denotes the set of real numbers. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathcal{P}$ denotes a probability measure. I denotes the identity matrix with compatible dimensions. The superscript "T" stands for matrix transposition. $\varepsilon\{\cdot\}$ denotes the expectation. $\|x\|$ refers to the Euclidean norm defined by

$$\|x\| = \sqrt{\sum_{i=1}^{n} x_i^2} \text{ for every } x = (x_1, x_2, \dots, x_n)^{\mathrm{T}} \in \mathbb{R}^n.$$

L_∞ stands for the set of all functions endowed with the essential supremum norm

$$|u| = \operatorname{ess\,sup}\bigl\{\|u\|,\ t \geqslant 0,\ u \in \mathbb{R}^q\bigr\} < \infty.$$

$a \vee b$ denotes the maximum of a and b. $\langle\cdot,\cdot\rangle$ stands for the inner product. E^+ denotes the pseudoinverse matrix of E and satisfies $EE^+E = E$.

2 Problem formulation

We consider the following stochastic singular nonlinear system:

$$\begin{aligned} E\,\mathrm{d}x(t) &= \bigl(Ax(t) + Hg\bigl(x(t)\bigr) + Bu(t) + C\omega(t)\bigr)\,\mathrm{d}t + Fx(t)\,\mathrm{d}w(t), \\ y(t) &= Dx(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in L_\infty$ is the control input, $\omega(t) \in \mathbb{R}^n$ is the disturbance input, $w(t)$ is a standard one-dimensional Brownian motion. $y(t) \in \mathbb{R}^m$ is the measurement output, $g(x(t))$: $\mathbb{R}^n \to \mathbb{R}^n$ is a continuous function and satisfies $g(0) = 0$. The matrix $E \in \mathbb{R}^{n\times n}$ may be singular, and $\operatorname{rank}(E) \leqslant n$. A, B, H, C, F and D are known constant matrices with appropriate dimensions.

Assumption 1. $\operatorname{rank}([E\ F]) = \operatorname{rank}(E)$.

Assumption 2. $g(x)$ satisfies the implicit function theorem and the following conditions:

$$\begin{aligned} \bigl\|g\bigl(x(t)\bigr)\bigr\|^2 &\leqslant K_0\bigl(1 + \bigl\|x(t)\bigr\|^2\bigr), \\ \bigl\|g\bigl(x_1(t)\bigr) - g\bigl(x_2(t)\bigr)\bigr\|^2 &\leqslant \rho_1\bigl\|x_1(t) - x_2(t)\bigr\|^2 \\ &\quad + \rho_2\bigl\langle x_1(t) - x_2(t),\, g\bigl(x_1(t)\bigr) - g\bigl(x_2(t)\bigr)\bigr\rangle, \end{aligned}$$

where $\rho_1 \in \mathbb{R}$, $\rho_2 \in \mathbb{R}$, $K_0 > 0$.

Remark 1. Under Assumption 1, the stochastic term $Fx(t)\,\mathrm{d}w(t)$ does not cause any changes on structure of system (1). That is, the Brownian motion does not affect the algebraic equations of singular system (1). Assumption 2 describes a broad family of nonlinear plants. By including much useful information of the nonlinear part, it extends the well-known Lipschitz property to a more general family of nonlinear systems.

For convenience, we shall introduce the following definitions:

Definition 1. (See [24].) A function $\gamma : \mathbb{R}^+ \to \mathbb{R}^+$ is a generalized $\mathcal{K}$-function if for all $s_1 > s_2 \geqslant 0$, it is continuous with $\gamma(0) = 0$ and satisfies

$$\begin{aligned} &\gamma(s_1) > \gamma(s_2), && \gamma(s_1) \neq 0, \\ &\gamma(s_1) = \gamma(s_2) = 0, && \gamma(s_1) = 0. \end{aligned}$$

Definition 2. (See [24].) A function $\beta : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is a generalized $\mathcal{KL}$-function if for each fixed $t \geqslant 0$, the function $\beta(s, t)$ is a generalized $\mathcal{K}$-function, and for each fixed $s \geqslant 0$ it is decreasing to zero on t as $t \to T$ for some finite constant $T < +\infty$.

Definition 3. (See [1].) System (1) is said to be finite-time stochastic input-to-state stability (FTSISS) if there exist a generalized $\mathcal{KL}$-function β and a generalized $\mathcal{K}$-function γ such that for any bounded input $\omega \in L_\infty$ and any initial state Ex_0, we have

$$\mathcal{P}\bigl\{\bigl\|x(t)\bigr\| \leqslant \beta\bigl(\|Ex_0\|,\, t - t_0\bigr) + \gamma\bigl(\bigl|\omega(t)\bigr|\bigr)\bigr\} \geqslant 1 - \rho \quad \forall t \geqslant t_0.$$

Remark 2. The main difference between stochastic input-to-state stability (SISS) [41] and finite-time stochastic input-to-state stability (FTSISS) is the finite-time convergence of β. For FTSISS, function $\beta(\|Ex_0\|,\, t-t_0) = 0$ when $t \geqslant t_0+T$. Different from normal systems, the initial conditions can cause the impulse behavior of singular systems. So we consider the FTSISS of system (1) with the admissible initial conditions.

Observer-based controller can be utilized when the system states are not completely accessible. The observer is given by

$$\begin{aligned} E\,\mathrm{d}\hat{x}(t) &= \bigl(A\hat{x}(t) + Bu(t) + G\bigl(\hat{y}(t) - y(t)\bigr)\bigr)\,\mathrm{d}t, \\ \hat{y}(t) &= D\hat{x}(t), \end{aligned} \tag{2}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the estimation of $x(t)$, and $G \in \mathbb{R}^{n\times m}$ is the observer gain.

State-estimation error dynamics is presented as $e(t) = x(t) - \hat{x}(t)$. Combining system (1) and (2), we can obtain the observer error dynamics as follows:

$$E\,\mathrm{d}e(t) = \bigl[(A+GD)e(t) + C\omega(t) + Hg\bigl(x(t)\bigr)\bigr]\,\mathrm{d}t + Fx(t)\,\mathrm{d}w(t). \tag{3}$$

For the controller gain matrix $K \in \mathbb{R}^{q\times n}$, the control input is given by

$$u(t) = K\hat{x}(t). \tag{4}$$

The closed-loop system can be modeled as follows:

$$\begin{aligned} E\,\mathrm{d}x(t) = {}& \bigl[(A+BK)x(t) + Hg\bigl(x(t)\bigr) - BKe(t) + C\omega(t)\bigr]\,\mathrm{d}t \\ & + Fx(t)\,\mathrm{d}w(t). \end{aligned} \tag{5}$$

Combining the above system and system (3), the augmented system is formed as follows:

$$\bar{E}\,\mathrm{d}\xi(t) = \bigl(\bar{A}\xi(t) + \bar{H}g\bigl(\xi(t)\bigr) + \bar{C}\omega(t)\bigr)\,\mathrm{d}t + \bar{F}\xi(t)\,\mathrm{d}w(t), \tag{6}$$

where

$$\bar{E} = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}, \qquad \bar{H} = \begin{bmatrix} H & 0 \\ H & 0 \end{bmatrix}, \qquad \bar{A} = \begin{bmatrix} A+BK & -BK \\ 0 & A+GD \end{bmatrix},$$
$$\bar{C} = \begin{bmatrix} C \\ C \end{bmatrix}, \qquad \bar{F} = \begin{bmatrix} F & 0 \\ F & 0 \end{bmatrix}, \qquad \xi(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}.$$

The objective of the present study is to explore observer-based control strategies for stochastic singular nonlinear systems with Brownian motions by computing the appropriate values of the gain matrices G and K.

Before giving the following lemmas, let us introduce an important operator. For the following stochastic system

$$\mathrm{d}x(t) = f\bigl(t, x(t), \omega(t)\bigr)\,\mathrm{d}t + \bar{g}\bigl(t, x(t), \omega(t)\bigr)\,\mathrm{d}w(t), \tag{7}$$

given $V(x,t) \in \mathbb{C}(\mathbb{R}^n \times [t_0,\infty) \to \mathbb{R}^+)$, define an operator $\mathcal{L}V(x(t),t)$ by

$$\mathcal{L}V\big(x(t),t\big) = \frac{\partial V(x(t),t)}{\partial t} + \frac{\partial V(x(t),t)}{\partial x} f(t,x(t),\omega(t)) + \frac{1}{2}\operatorname{trace}\bigg[\bar{g}^{\mathrm{T}}\big(t,x(t),\omega(t)\big)\frac{\partial^2 V(x(t),t)}{\partial x^2}\bar{g}\big(t,x(t),\omega(t)\big)\bigg].$$

Lemma 1. (*See* [1].) *For any process $Y(t)$, $\varepsilon(Y(t))$ is locally absolutely continuous and $0 \leqslant \varepsilon(Y(t)) < \infty$ for any $t \geqslant t_0$, if there exists a continuous convex function $h(r) \in \mathcal{K}$ and a constant $c_1 > 0$ such that*

$$\mathcal{L}Y(t) \leqslant -h\big(Y(t)\big) - c_1, \quad \textit{a.s. } Y(t) \neq 0,$$
$$\mathcal{L}Y(t) = 0, \quad Y(t) = 0,$$

then there exists a generalized $\mathcal{KL}$ function β satisfying

$$\varepsilon\big(Y(t)\big) \leqslant \beta\big(\varepsilon(Y_0),\, t - t_0\big), \quad t \geqslant t_0. \tag{8}$$

Lemma 2. (*See* [33].) *The pair (E,A) is impulse-free if and only if A_4 is nonsingular, where there are nonsingular matrices $\bar{G}$ and Q such that*

$$\bar{G}EQ = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \qquad \bar{G}AQ = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}.$$

Lemma 3. (*See* [20].) *Let c_0, d_0 be positive real numbers and $\gamma_0(x,y) > 0$ be a real-valued function, then*

$$|x|^{c_0}|y|^{d_0} \leqslant \frac{c_0}{c_0+d_0}\gamma_0(x,y)|x|^{c_0+d_0} + \frac{d_0}{c_0+d_0}\gamma_0^{-c_0/d_0}(x,y)|y|^{c_0+d_0}.$$

3 Main results

This section is concerned with the design of observer-based controller for a class of stochastic singular nonlinear systems. The observer gain and controller gain are obtained by an LMI approach such that the augmented system (6) is impulse free and FTSISS. We first present a sufficient condition of FTSISS. We denote $x(t) = x$, $\omega(t) = \omega$ and $\xi(t) = \xi$ for simplicity.

Theorem 1. *System* (7) *is finite-time stochastic input-to-state stable* (*FTSISS*) *if there exists a function $V(x,t) \in \mathbb{C}(\mathbb{R}^n \times [t_0,\infty) \to \mathbb{R}^+)$, functions α_1, α_2, and $\varphi \in \mathcal{K}_\infty$ and a scalar $0 < \eta < 1$ such that*

$$\alpha_1\big(\|x\|\big) \leqslant V(x,t) \leqslant \alpha_2\big(\|x\|\big), \tag{9}$$

$$\mathcal{L}V(x,t) \leqslant -\lambda_1\big(V(x,t)\big)^\eta + \varphi\big(|u|\big), \quad x \in \mathbb{R}^n \setminus \{0\}, \tag{10}$$

$$x = 0 \quad \Longrightarrow \quad \mathcal{L}V(x,t) = 0.$$

Proof. Let $\tau_0 \in [t_0, \infty)$, a time at which the trajectory enters the set

$$\Phi = \{x \in \mathbb{R}^n,\ V(x,t) \leqslant \chi(|u|)\},$$

where $\chi(|u|) = (((1+\sigma)/(\lambda_1 - \lambda))\varphi(|u|))^{1/\eta}$, $\lambda_1 > \lambda$. When $x_0 = 0$, the conclusion can be drawn directly. In the following analysis, x_0 is divided into two parts.

Case 1. $x_0 \in \Phi^c \setminus \{0\}$, where Φ^c denotes the complementary set of Φ. In this case, for any $t \in [t_0, \tau_0]$,

$$\big(V(x,t)\big)^\eta > \big(\chi(|u|)\big)^\eta.$$

Using the above inequality, inequality (8) and $\chi(|u|)$, we obtain

$$-\lambda_1\big(V(x,t)\big)^\eta \leqslant -\lambda\big(V(x,t)\big)^\eta - (1+\sigma)\varphi(|u|),$$

then

$$\mathcal{L}V(x,t) \leqslant -\lambda_1\big(V(x,t)\big)^\eta + \varphi(|u|) \leqslant -\lambda\big(V(x,t)\big)^\eta - \sigma\varphi(|u|).$$

Because the perturbation input u is bounded function, then there exists constant α such that

$$\mathcal{L}V(x,t) \leqslant -\lambda\big(V(x,t)\big)^\eta - \alpha.$$

Then from Lemma 1 and above expressions there exists a generalized $\mathcal{KL}$ function $\tilde{\beta}$ satisfying the following condition:

$$\varepsilon\big(V(x,t)\big) \leqslant \tilde{\beta}(V_0,\, t - t_0) \quad \forall t \in [t_0, \tau_0].$$

Applying the Chebyshev's inequality [39], we have

$$\mathcal{P}\big\{V(x,t) \geqslant \hat{\beta}(V_0,\, t-t_0)\big\} \leqslant \frac{\tilde{\beta}(V_0,\, t-t_0)}{\hat{\beta}(V_0,\, t-t_0)} = \rho,$$

where $\hat{\beta} = \tilde{\beta}/\rho \in \mathcal{KL}$, $\rho \in (0,1)$.

Thus

$$\mathcal{P}\big\{V(x,t) < \hat{\beta}(V_0,\, t-t_0)\big\} \geqslant 1-\rho \quad \forall t \in [t_0, \tau_0].$$

Denote $\beta = \alpha_1^{-1}(\hat{\beta}(\alpha_2(\cdot)))$, we have

$$\mathcal{P}\big\{\|x\| < \beta(\|x_0\|, t-t_0)\big\} \geqslant 1-\rho \quad \forall t \in [t_0, \tau_0]. \tag{11}$$

We define a function $\psi(V) = \int_0^V 1/\lambda(v(x,t))^\eta \,\mathrm{d}v$, according to the Itô formula and (9), it can be verified that

$$\mathcal{L}(\psi(V)) \leqslant \frac{\mathcal{L}V}{\lambda(V(x,t))^\eta} \leqslant \frac{-\lambda(V(x,t))^\eta}{\lambda(V(x,t))^\eta} \leqslant -1.$$

Integrating both sides of the above inequality from 0 to T, then

$$\psi\big(V(T)\big) - \psi(V_0) \leqslant -T,$$

that is, $T \leqslant \psi(V_0) < \infty$.

Now let us consider the interval $t \in (\tau_0, \infty)$, that is,

$$\varepsilon\big(V(x,t)\big) \leqslant \chi\big(|u|\big) \quad \forall t > \tau_0.$$

By the Chebyshev's inequality, for any class $\mathcal{K}_\infty$ function δ, it follows that

$$\mathcal{P}\Big\{\sup_{t>\tau} V(x,t) \geqslant \delta\big(\chi\big(|u|\big)\big)\Big\} \leqslant \frac{\chi(|u|)}{\delta(\chi(|u|))} \leqslant \rho',$$

where ρ' can be made arbitrarily small by an appropriate choice of $\delta \in \mathcal{K}_\infty$. Then we have

$$\mathcal{P}\big\{\|x\| < \alpha_2^{-1}\big(\delta\big(\chi\big(|u|\big)\big)\big)\big\} \geqslant 1-\rho', \quad t > \tau_0.$$

Then combined with (10), it is verified that for any $x_0 \in \Phi^c \setminus \{0\}$ and $t \geqslant t_0$,

$$\mathcal{P}\big\{\|x\| < \beta\big(\|x_0\|, t-t_0\big) + \gamma\big(|u|\big)\big\} \geqslant \max\{1-\rho, 1-\rho'\} = 1-\rho''. \tag{12}$$

Case 2. $x_0 \in \Phi \setminus \{0\}$, in this case $\tau_0 = t_0$ a.s.

When $t > t_0$, following the proof of Case 1,

$$\mathcal{P}\big\{\|x\| < \beta\big(\|x_0\|, t-t_0\big) + \gamma\big(|u|\big)\big\} \geqslant \mathcal{P}\big\{\|x\| < \gamma\big(|u|\big)\big\} \geqslant 1-\rho''.$$

At the same time $t = t_0$, by the definition of γ, the definition of the set Φ and inequalities (8) and (9), we obtain

$$\mathcal{P}\big\{\|x_0\| < \beta\big(\|x_0\|, 0\big) + \gamma\big(|u|\big)\big\} \geqslant \mathcal{P}\big\{\|x_0\| < \gamma\big(|u|\big)\big\},$$

which includes

$$\mathcal{P}\big\{\|x_0\| < \beta\big(\|x_0\|, 0\big) + \gamma\big(|u|\big)\big\} = 1.$$

Thus, by (11) and (12), we have for any $x_0 \in \Phi \setminus \{0\}$ and $t \geqslant t_0$,

$$\mathcal{P}\big\{\|x\| < \beta\big(\|x_0\|, t-t_0\big) + \gamma\big(|u|\big)\big\} \geqslant 1-\rho''.$$

Then for any $x_0 \in \mathbb{R}^n \setminus \{0\}$ and $t \geqslant t_0$,

$$\mathcal{P}\big\{\|x\| < \beta\big(\|x_0\|, t-t_0\big) + \gamma\big(|u|\big)\big\} \geqslant 1-\rho''.$$

According to Definition 3, system (7) is FTSISS. This completes the proof. □

Remark 3. Finite-time stability means finite-time convergence. SISS characterizes the effects of the input on the state. The concept of FTSISS combines finite-time stability and the SISS. It is our firm belief that FTSISS will play a role in finite-time control. So in Theorem 1, we give a novel sufficient condition of FTSISS for stochastic nonlinear systems.

Now we use the results of Theorem 1 to give the important result for stochastic singular nonlinear system (1).

Theorem 2. *The stochastic singular nonlinear system* (1) *with* $u(t) = 0$ *is impulse-free and FTSISS if there exist matrices* P *and* $\bar{P} > 0$ *and a scalar* $\tau > 0$ *such that the following inequalities hold*:

$$\bar{P} - 8\tau|\rho_1|I - 4\tau\rho_2^2 I > 0, \tag{13}$$

$$E^{\mathrm{T}}P = P^{\mathrm{T}}E \geqslant 0,$$

$$\Omega = \begin{bmatrix} \Omega_0 & P^{\mathrm{T}}C & P^{\mathrm{T}}H + \tau\rho_2 I \\ * & -\tau I & 0 \\ * & * & -2\tau I \end{bmatrix} < 0, \tag{14}$$

where $\Omega_0 = P^{\mathrm{T}}A + A^{\mathrm{T}}P + F^{\mathrm{T}}(E^+)^{\mathrm{T}}E^{\mathrm{T}}PE^+F + 2\tau\rho_1 I + \bar{P}$.

Proof. We first show that the pair (E, A) is impulse-free. There are nonsingular matrices $\bar{G}$ and Q such that

$$\bar{G}EQ = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \qquad \bar{G}AQ = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \qquad \bar{G}^{-\mathrm{T}}PQ = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}.$$

From (13) and (14) we have $P^{\mathrm{T}}A + A^{\mathrm{T}}P < 0$. Then

$$\begin{bmatrix} * & * \\ * & P_{22}^{\mathrm{T}}A_4 + A_4^{\mathrm{T}}P_{22} \end{bmatrix} < 0.$$

It is easy to see that $P_{22}^{\mathrm{T}}A_4 + A_4^{\mathrm{T}}P_{22} < 0$, which implies that A_4 is nonsingular. According to Lemma 2, it is easy to find that the pair (E, A) is impulse-free. Then there exist nonsingular matrices $M = [M_1\ \ M_2]^{\mathrm{T}}$ and $N = [N_1\ \ N_2]$ with appropriate dimensions such that the following standard decompositions hold:

$$\tilde{E} = MEN = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \qquad \tilde{A} = MAN = \begin{bmatrix} \tilde{A}_1 & 0 \\ 0 & I \end{bmatrix},$$
$$\tilde{P} = M^{-\mathrm{T}}PN = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix}.$$

Then system (1) with $u = 0$ and $\omega = 0$ can be transformed into the following form:

$$\begin{aligned} \mathrm{d}x_1 &= \big(\tilde{A}_1 x_1 + M_1 H g\big(N_1 x_1 + N_2 x_2\big)\big)\,\mathrm{d}t + (C_1 x_1 + C_2 x_2)\,\mathrm{d}w, \\ 0 &= x_2 + M_2 H g(N_1 x_1 + N_2 x_2), \end{aligned} \tag{15}$$

where $N^{-1}x = [x_1^{\mathrm{T}}\ \ x_2^{\mathrm{T}}]^{\mathrm{T}}$.

By Assumption 2, we can find that the function $M_2 H g(N_1 x_1 + N_2 x_2)$ satisfies the implicit function theorem. Then the solution of system (1) exists .

Next, we show that system (1) is FTSISS. Choosing a Lyapunov function

$$V(x_1) = x_1^{\mathrm{T}} P_1 x_1 = x^{\mathrm{T}}E^{\mathrm{T}}Px,$$

where $P_1 > 0$, we obtain the following inequality:

$$\lambda_{\min}(P_1)\|x_1\|^2 \leqslant V(x_1) \leqslant \lambda_{\max}(P_1)\|x_1\|^2.$$

Using the Itô formula, we obtain

$$\begin{aligned}
\mathcal{L}V &= x^{\mathrm{T}}P^{\mathrm{T}}\big(Ax + C\omega + Hg(x)\big) + \big(Ax + C\omega + Hg(x)\big)^{\mathrm{T}}Px \\
&\quad + x^{\mathrm{T}}F^{\mathrm{T}}\big(E^{+}\big)^{\mathrm{T}}E^{\mathrm{T}}PE^{+}Fx \\
&= x^{\mathrm{T}}\big(P^{\mathrm{T}}A + A^{\mathrm{T}}P + F^{\mathrm{T}}\big(E^{+}\big)^{\mathrm{T}}E^{\mathrm{T}}PE^{+}F\big)x + x^{\mathrm{T}}P^{\mathrm{T}}C\omega \\
&\quad + \omega^{\mathrm{T}}C^{\mathrm{T}}Px + x^{\mathrm{T}}P^{\mathrm{T}}Hg(x) + g^{\mathrm{T}}(x)H^{\mathrm{T}}Px.
\end{aligned}$$

It follows from Assumption 2 that for any positive scalar $\tau > 0$, it can be discerned that

$$2\tau\big(\rho_1 x^{\mathrm{T}}x + \rho_2 g^{\mathrm{T}}(x)x - g^{\mathrm{T}}(x)g(x)\big) \geqslant 0,$$

then

$$\begin{aligned}
\mathcal{L}V &\leqslant x^{\mathrm{T}}P^{\mathrm{T}}C\omega + \omega^{\mathrm{T}}C^{\mathrm{T}}Px + x^{\mathrm{T}}\big(P^{\mathrm{T}}H + \tau\rho_2 I\big)g(x) \\
&\quad + x^{\mathrm{T}}\big(P^{\mathrm{T}}A + A^{\mathrm{T}}P + F^{\mathrm{T}}\big(E^{+}\big)^{\mathrm{T}}E^{\mathrm{T}}PE^{+}F + 2\tau\rho_1 I + \bar{P}\big)x \\
&\quad + g^{\mathrm{T}}(x)\big(H^{\mathrm{T}}P + \tau\rho_2 I\big)x - 2\tau g^{\mathrm{T}}(x)g(x) - \tau\omega^{\mathrm{T}}\omega - x^{\mathrm{T}}\bar{P}x + \tau\omega^{\mathrm{T}}\omega \\
&= \zeta^{\mathrm{T}}\Omega\zeta - x^{\mathrm{T}}\bar{P}x + \tau_1\omega^{\mathrm{T}}\omega,
\end{aligned}$$

where $\bar{P} > 0$, $\zeta = [x^{\mathrm{T}}\ \ \omega^{\mathrm{T}}\ \ g^{\mathrm{T}}(x)]^{\mathrm{T}}$.

By (14), we know $\Omega < 0$, then $\mathcal{L}V \leqslant -x^{\mathrm{T}}\bar{P}x + \tau\omega^{\mathrm{T}}\omega$. We can find a scalar $\bar{\lambda} > 0$ such that for $x \neq 0$,

$$\mathcal{L}V \leqslant -\bar{\lambda}x^{\mathrm{T}}x + \tau\omega^{\mathrm{T}}\omega. \tag{16}$$

Let $0 < d < 1$. According to Lemma 3, we obtain

$$\|x\|^{1+d} \leqslant \frac{1-d}{2}\big(ag(x)\operatorname{sgn}\big(g(x)\big) + b\big)^{1+d} + \frac{1+d}{2}\frac{\|x\|^2}{(ag(x)\operatorname{sgn}(g(x)) + b)^{1-d}}, \tag{17}$$

where $a > 0$ and $b > 0$.

Using Assumption 2, (17) can be changed into the following form:

$$\begin{aligned}
\|x\|^{1+d} &\leqslant (1-d)\big(a^{1+d}\big(K_0\big(1 + \|x\|^2\big)\big)^{(1+d)/2} + b^{1+d}\big) + \frac{1+d}{2}\frac{\|x\|^2}{b^{1-d}} \\
&\leqslant 2(1-d)a^{1+d}K_0^{(1+d)/2} + 2(1-d)a^{1+d}K_0^{(1+d)/2}\|x\|^{1+d} \\
&\quad + (1-d)b^{1+d} + \frac{1+d}{2}\frac{\|\xi\|^2}{b^{1-d}}.
\end{aligned}$$

Let $a < (1/(2(1-d)K_0^{(1+d)/2}))^{1/(1+d)}$, then $c = 2(1-d)a^{1+d}K_0^{(1+d)/2} < 1$. We obtain

$$\|x\|^{1+d} \leqslant \frac{c_1}{1-c} + \frac{1+d}{2(1-c)}\frac{\|x\|^2}{b^{1-d}}, \tag{18}$$

where $c_1 = 2(1-d)a^{1+d}K_0^{(1+d)/2} + (1-d)b^{1+d}$.

By (16) and (18), we have

$$\begin{aligned}
&\mathcal{L}V + \mu V^{(1+d)/2}(x_1)\\
&\quad\leqslant -\bar{\lambda}x^{\mathrm{T}}x + \bar{\gamma}^2 u^{\mathrm{T}}u - k_0\|x\|^{1+d} + k_0\|x\|^{1+d} + \mu\bar{\lambda}_0\|x\|^{1+d}\\
&\quad\leqslant -\left(\bar{\lambda} - k_0\frac{1+d}{2(1-c)b^{1-d}}\right)\|x\|^2 - (k_0 - \mu\bar{\lambda}_0)\|x\|^{1+d} + \bar{\gamma}^2\omega^{\mathrm{T}}\omega + \frac{k_0 c_1}{1-c},
\end{aligned}$$

where $\mu > 0$.

Let $k_0 < (2\bar{\lambda}(1-c)b^{1-d})/(1+d)$ and $\mu < k_0/\bar{\lambda}_0$. Choose appropriate k_0 and μ, then

$$\mathcal{L}V \leqslant -\mu V^{(1+d)/2}(x_1) + \bar{\gamma}^2\omega^{\mathrm{T}}\omega.$$

By Theorem 1, for state variable x_1, the following condition is satisfied:

$$\mathcal{P}\bigl\{\|x_1\| \leqslant \beta\bigl(\|x_{10}\|, t - t_0\bigr) + \gamma\bigl(|\omega|\bigr)\bigr\} \geqslant 1 - \rho. \tag{19}$$

On the other hand, according to (14), we have

$$\begin{bmatrix} \varOmega & P^{\mathrm{T}}H + \tau\rho_2 I\\ * & -2\tau I\end{bmatrix} < 0,$$

then we can find a scalar $p > 0$ such that

$$\begin{bmatrix} P^{\mathrm{T}}\bar{A} + \bar{A}^{\mathrm{T}}P + pI & P^{\mathrm{T}}H + \tau\rho_2 I\\ * & -2\tau I\end{bmatrix} < 0. \tag{20}$$

By (20), the following inequality holds:

$$\begin{bmatrix} \tilde{P}^{\mathrm{T}}\tilde{A} + \tilde{A}^{\mathrm{T}}\tilde{P} + pN^{\mathrm{T}}N & \tilde{P}^{\mathrm{T}}MH + \tau\rho_2 N^{\mathrm{T}}\\ * & -2\tau I\end{bmatrix} < 0. \tag{21}$$

Using (15) and (21), we obtain the inequality as follows:

$$\begin{bmatrix} P_4 + P_4^{\mathrm{T}} + pN_2^{\mathrm{T}}N_2 & P_4^{\mathrm{T}}M_2H + \tau\rho_2 N_2^{\mathrm{T}}\\ * & -2\tau I\end{bmatrix} < 0. \tag{22}$$

Pre-multiplying and post-multiplying (22) by $\varGamma = \left[\begin{smallmatrix} H^{\mathrm{T}}M_2^{\mathrm{T}} & -I\\ 0 & I\end{smallmatrix}\right]$ and $\varGamma^T$, respectively, we have

$$pH^{\mathrm{T}}M_2^T N_2^{\mathrm{T}}N_2M_2H - \tau\rho_2 N_2M_2H - \tau\rho_2 H^{\mathrm{T}}M_2^{\mathrm{T}}N_2^{\mathrm{T}} - 2\tau I \leqslant 0,$$

then

$$\begin{aligned}
px_2^T N_2^{\mathrm{T}}N_2x_2 &= pg^{\mathrm{T}}(x)H^{\mathrm{T}}M_2^T N_2^{\mathrm{T}}N_2M_2Hg(x)\\
&\leqslant \tau\rho_2 g^{\mathrm{T}}(x)H^{\mathrm{T}}M_2^T N_2^T g(x) + \tau\rho_2 g^{\mathrm{T}}(x)N_2M_2Hg(x) + 2\tau g^{\mathrm{T}}(x)g(x)\\
&= -\tau\rho_2 x_2^{\mathrm{T}}N_2^T g(x) - \tau\rho_2 g^{\mathrm{T}}(x)N_2x_2 + 2\tau g^{\mathrm{T}}(x)g(x)\\
&\leqslant \tau\rho_2^2 x_2^T N_2^{\mathrm{T}}N_2x_2 + 3\tau g^{\mathrm{T}}(x)g(x).
\end{aligned} \tag{23}$$

By Assumption 2, it is easy to obtain the following inequality:

$$\begin{aligned} g^{\mathrm{T}}(x)g(x) &\leqslant \rho_1 x^{\mathrm{T}}x + \rho_2 g^{\mathrm{T}}(x)x \\ &\leqslant \left(|\rho_1| + \delta_1^{-1}\rho_2^2\right)x^{\mathrm{T}}x + \frac{\delta_1}{4}g^{\mathrm{T}}(x)g(x) \\ &\leqslant \bar{\delta}_1 x_1^T N_1^{\mathrm{T}} N_1 x_1 + \bar{\delta}_2 x_2^T N_2^{\mathrm{T}} N_2 x_2 + \frac{\delta_1}{4}g^{\mathrm{T}}(x)g(x), \end{aligned} \tag{24}$$

where $\bar{\delta}_1 = (|\rho_1| + \delta_1^{-1}\rho_2^2)(1+\delta_2)$,$\bar{\delta}_2 = (|\rho_1| + \delta_1^{-1}\rho_2^2)(1+\delta_2^{-1})$.

Given appropriate scalars δ_1 and δ_2. Using inequalities (13), (23) and (24), we can find functions $\beta_1 \in \mathcal{KL}$ and $\gamma_1 \in \mathcal{K}$ such that the following inequality holds:

$$\varepsilon\left(\|x_2\|\right) \leqslant K_1 \varepsilon\left(\|x_1\|\right) + \rho_3 \varepsilon\left(\|\omega\|\right).$$

When the condition $\varepsilon(\|x_1\|) \leqslant \beta(\|x_{10}\|, t-t_0) + \gamma(|\omega|)$ holds, we have

$$\varepsilon\left(\|x_2\|\right) \leqslant \beta_1\left(\|x_{10}\|, t-t_0\right) + \gamma_1\left(|\omega|\right).$$

Applying the Chebyshev's inequality, we have

$$\mathcal{P}\left\{\|x_2\| \geqslant \tilde{\beta}_1\left(\|x_{10}\|, t-t_0\right) + \tilde{\gamma}_1\left(|u|\right)\right\} \leqslant \tilde{\rho},$$

where $\tilde{\beta}_1 = \beta_1/\tilde{\rho} \in \mathcal{KL}$, $\tilde{\rho} \in (0,1)$. Thus

$$\mathcal{P}\left\{\|x_2\| \leqslant \tilde{\beta}_1\left(\|x_{10}\|, t-t_0\right) + \tilde{\gamma}_1\left(|\omega|\right)\right\} \geqslant 1 - \tilde{\rho}. \tag{25}$$

According to (19), (25) and Theorem 1, system (1) is FTSISS. This completes the proof. □

Sufficient conditions in term of LMIs, which allow us to compute the gain matrices G and K for the observer-based control design for stochastic singular nonlinear systems, will be present in the next theorem, which utilizes the results of Theorem 2.

Theorem 3. *Consider the augmented system* (6) *constituted of stochastic singular nonlinear system* (1), *observer* (2) *and controller* (4). *Let there exist matrices* $P_1 > 0$, P_k, P_G, P_2, $P_{01} > 0$ *and* $P_{02} > 0$ *and a scalar* $\tau > 0$ *such that the following inequalities hold:*

$$E^{\mathrm{T}}P_1 = P_1^T E \geqslant 0, \tag{26}$$

$$E^{\mathrm{T}}P_2 = P_2^T E \geqslant 0, \tag{27}$$

$$\begin{bmatrix} P_{01} - \Upsilon & 0 \\ 0 & P_{02} - \Upsilon \end{bmatrix} > 0, \tag{28}$$

$$\begin{bmatrix} \Psi_1 & -P_k & P_1^{\mathrm{T}}C & \Psi_3 & 0 \\ * & \Psi_2 & P_2^{\mathrm{T}}C & P_2^{\mathrm{T}}H & \tau\rho_2 I \\ * & * & -\tau I & 0 & 0 \\ * & * & * & -2\tau I & 0 \\ * & * & * & * & -2\tau I \end{bmatrix} < 0, \tag{29}$$

where

$$\begin{aligned}
\Upsilon &= 8\tau|\rho_1|I + 4\tau\rho_2^2 I,\\
\Psi_1 &= P_1^T A + A^{\mathrm{T}} P_1 + F^{\mathrm{T}}(E^+)^{\mathrm{T}} E^{\mathrm{T}} \tilde{P}_1 E^+ F + P_k^{\mathrm{T}} + P_k + P_{01} + 2\tau\rho_1 I,\\
\Psi_2 &= P_2^{\mathrm{T}} A + A^{\mathrm{T}} P_2 + P_G^{\mathrm{T}} D + D^{\mathrm{T}} P_G + P_{02} + 2\tau\rho_1 I,\\
\Psi_3 &= P_1^T H + \tau\rho_2 I.
\end{aligned}$$

Then the closed-loop system (6) *is impulse-free and FTSISS, and the gains are given by*

$$G = P_2^{-\mathrm{T}} P_G^{\mathrm{T}}, \qquad K = \left(P_1^{\mathrm{T}} B\right)^{-1} P_k.$$

Proof. Define

$$P = \begin{bmatrix} P_1 & 0\\ 0 & P_2 \end{bmatrix}, \qquad P_k = P_1^{\mathrm{T}} B K, \qquad P_G^{\mathrm{T}} = P_2^{\mathrm{T}} G. \tag{30}$$

It is easy to prove that LMI (29) guarantees inequality (14) holds. Therefore, (26)–(29) indicate that the closed-loop system is impulse-free and FTSISS. Subsequently, the parameters of the controller and observer can be solved from (30). This completes the proof. □

Remark 4. For linear or nonlinear systems, the conventional methods use state feedback to ensure convergence of states of systems when studying the control schemes. However, sometime these methods cannot be practically used, for instance, when the states of systems are not available for feedback. And all states of the measurement system require additional sensors and additional hardware for amplification and calibration, which limits the applicability of the solution in real-world scenarios. Our work employs estimated states, which can be obtained by observer for the control purpose. It is more practical approach.

4 Simulation examples

Example 1. Consider the stochastic singular nonlinear system (1) with the following parameters:

$$E = \begin{bmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0 \end{bmatrix}, \qquad A = \begin{bmatrix} 2 & -3.6 & -7\\ 6.1 & -3 & 3.2\\ 2 & -3.5 & -6.4 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0\\ 1 & 2\\ 0 & 0 \end{bmatrix},$$

$$H = \begin{bmatrix} -0.1 & 0 & -0.1\\ 0 & 0.8 & -0.3\\ -0.4 & -0.2 & -0.1 \end{bmatrix}, \qquad C = \begin{bmatrix} -0.1\\ -0.2\\ 0.1 \end{bmatrix}, \qquad D = \begin{bmatrix} 0.5 & -0.2\\ -0.2 & 0.1\\ 0.1 & -0.25 \end{bmatrix}^{\mathrm{T}},$$

$$F = \begin{bmatrix} 0.3 & -0.2 & 0.1\\ 0 & 0.1 & -0.2\\ 0 & 0 & 0 \end{bmatrix}, \qquad g(x) = \begin{bmatrix} -x_1(x_1^2 + x_2^2)\\ -x_2(x_1^2 + x_2^2) \end{bmatrix},$$

where the initial states are $x(0) = [0.5\ \ 1\ \ 0]^{\mathrm{T}}$.

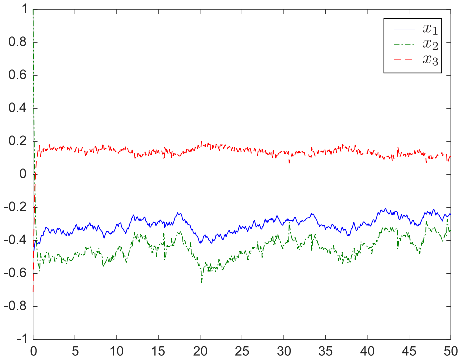

Figure 1. The responses of the uncontrolled system (1).

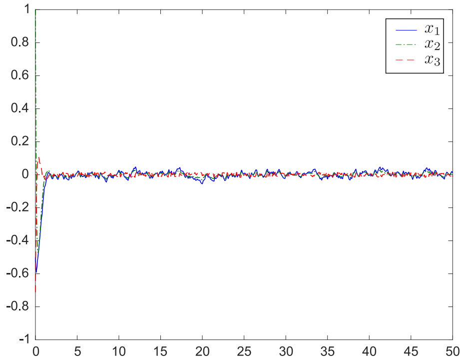

Figure 2. The responses of the controlled system (5).

For $\rho_1 = -0.3$, $\rho_2 = 0.1$, using Theorem 3 and LMI control toolbox in Matlab, the controller and observer gains can be obtain as follows:

$$K = \begin{bmatrix} -1.6879 & -2.1946 & 0.7447 \\ -1.7202 & -1.8858 & 0.5662 \end{bmatrix},$$
$$G = \begin{bmatrix} -22.3156 & 30.6459 & -64.5735 \\ -34.8675 & 53.2088 & -94.1230 \end{bmatrix}^{\mathrm{T}}.$$

Under the observer and controller with the gains above, the responses of the uncontrolled system (1) with $u(t) = 0$ and the responses of system (5) are illustrated in Figs. 1 and 2, respectively. It can be observed from Figs. 1 and 2 that system (1) with Brownian motions has been stabilized by the observer-based controller.

Example 2. Let us consider the following nonlinear stochastic singular bio-economic model:

$$\begin{aligned} \mathrm{d}x(t) &= \bigl(az(t) - bx(t) + b_1u(t) + c_1v(t)\bigr)\,\mathrm{d}t + \sigma z(t)\,\mathrm{d}\omega(t), \\ \mathrm{d}z(t) &= \bigl(\delta x(t) - \beta z^2(t) - z(t)E_0(t) + b_2u(t) + c_2v(t)\bigr)\,\mathrm{d}t, \\ 0 &= E_0(t)\bigl(pz(t) - c\bigr) - m + b_3u(t) + c_3v(t), \end{aligned} \tag{31}$$

where the concepts of model parameters can be defined in [32]. The values of parameters are as follows: $a = 0.2$, $b = 2$, $b_1 = 0.2$, $c_1 = 0.2$, $\sigma = 0.06$, $\delta = 0.05$, $b_2 = -0.1$, $c_2 = 0.1$, $p = 1$, $c = 30$, $b_3 = 0.3$, $c_3 = -0.1$.

For $\rho_1 = -0.2$, $\rho_2 = 0.3$ and the initial states $x(0) = [1\ \ 2\ \ 0.1]^{\mathrm{T}}$, using Theorem 3 and LMI control toolbox in Matlab, the controller and observer gains can be obtain as follows:

$$K = \begin{bmatrix} 1.4428 & 0.9827 & -0.0015 \end{bmatrix},$$
$$G = \begin{bmatrix} 5.7173 & 32.5526 & 34.5364 \end{bmatrix}^{\mathrm{T}}.$$

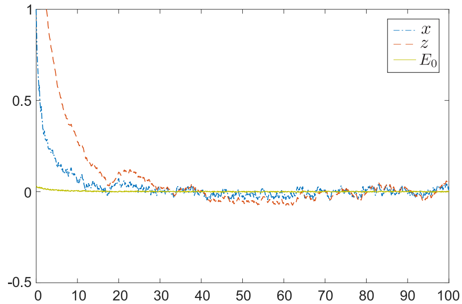

Figure 3. The responses of the controlled system (31).

Under the observer and controller with the gains above, the responses of system (31) are illustrated in Fig. 3. As can be concluded from the simulation results, the proposed controller has a good tracking performance in this paper.

5 Conclusions

In this paper, the observer-based controller for a class of singular nonlinear systems is investigated. For stochastic nonlinear systems with Brownian motions, sufficient conditions of the finite-time stochastic input-to-state stability has been proposed. Based on the sufficient conditions, the observer-based control technique has been proposed for a class of stochastic singular nonlinear systems, where the control gain and observer gain can be obtained by solving linear matrix inequalities. In some practical systems, state-feedback controllers cannot be used when systems states are not completely known. Based on this view, an observer-based controller is designed in this paper. Simulation examples have been given to illustrate the effectiveness of the proposed approach. It is worth pointing out that the proposed design methods are significantly extended to the case for stochastic singular nonlinear systems with multiple Brownian motions, and the observer-based controller designed for general stochastic singular nonlinear systems will be an interesting work to be considered further.

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